

# 26

**MATHEMATICAL ECONOMICS IN THE EXPLANATION OF ECONOMIC GROWTH IN ECONOMIES WITH ENDOGENOUS AND EXOGENOUS TECHNOLOGICAL CHANGE.**

**LA ECONOMÍA MATEMÁTICA EN LA EXPLICACIÓN DEL CRECIMIENTO ECONÓMICO EN ECONOMÍAS CON CAMBIO TECNOLÓGICO ENDÓGENO Y EXÓGENO.**

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## ABSTRACT

Economic growth is a function of the interactions between the different productive factors framed in the economic policy of an economy, in particular, it can be expressed in terms of labour force, productive resources (land, capital) and technology, among others. The present work pretends to approximate a model to explain the economic growth in developing economies, for which a model is proposed that explains this growth in function of the referred factors; then production is proposed in function of capital and work and two models are adjusted, one with exogenous technological change and the other that involves technological change in an endogenous manner.

The model is developed with a production function with constant substitution elasticity so that it is applicable to both developed and developing economies, since it is to be expected that in developed economies the substitution elasticity is unitary, which would lead to a Cobb-Douglas-type production function, but it is very probable that in incipient economies the function with constant substitution elasticity better reflects the relationship between production factors and economic growth. The research allows the development of the corresponding mathematical model in each case, the economic and mathematical foundations of each model are presented and validated according to economic theories. The behaviour of variables such as savings, investment, income, consumption, capital and their relationships in each model is analysed.

## RESUMEN

El crecimiento económico es una función de las interacciones entre los diferentes factores productivos enmarcados en la política económica de una economía, en particular, puede expresarse en términos de mano de obra, recursos productivos (tierra, capital) y tecnología, entre otros. El presente trabajo pretende aproximarse a un modelo para explicar el crecimiento económico en las economías en desarrollo, para lo cual se propone un modelo que explica este crecimiento en función de los factores referidos; luego se propone la producción en función del capital y el trabajo y se ajustan dos modelos, uno con el cambio tecnológico exógeno y otro que implica el cambio tecnológico de manera endógena. El modelo se desarrolla con una función de producción con elasticidad de sustitución constante de manera que es aplicable tanto a economías desarrolladas como en desarrollo, ya que es de esperar que en las economías desarrolladas la elasticidad de sustitución sea unitaria, lo que llevaría a una función de producción tipo Cobb-Douglas, pero es muy probable que en economías incipientes la función con elasticidad de sustitución constante refleje mejor la relación entre los factores de producción y el crecimiento económico. La investigación permite desarrollar el correspondiente modelo matemático en cada caso, se presentan y validan los fundamentos económicos y matemáticos de cada modelo según las teorías económicas. Se analiza el comportamiento de variables como el ahorro, la inversión, la renta, el consumo, el capital y sus relaciones en cada modelo.

## INTRODUCTION

Economic growth is understood as a dynamic process in which each economy establishes interactions between the different productive factors that, together with economic policies, allows the generation of a greater quantity of goods and services produced, which improves the wellbeing of the population. [1, 2].

A model of economic growth is based on economic theory to establish basic fundamental assumptions that allow proposing an interaction between the factors of production in order to explain the determinants of economic growth [3, 4]

Based on exogenous and endogenous growth theories, different models have been proposed to explain economic growth including the human factor, capital accumulation and technological change among other factors of production. In this sense, Paul Romer [5, 6], Lucas [7], Aghion and Howitt [8], Grosman and Hellpman [9], Guelllec and Ralle [10] and Gaviria [11] among others, present theoretical works that explain endogenous growth.

Lucas [7] presents three models of economic growth to describe the production of a country based on its levels of physical and human capital and its level of technological acquis, using a production function of Cobb-Douglas. In the first model emphasizes the accumulation of human capital and technological change as determinants of economic growth, considers, like Solow, that the rate of change of technology is exogenous and determines the trajectories that per capita consumption should follow (control variable) and the stock of capital (state variable) in order to maximize utility over time.

In the second model, it assumes that technological change is endogenous and uses the approaches of Usawa [12] to measure its rate of growth. Its purpose is to find the trajectories that must be followed by the variables of control, per capita consumption and effort destined to production, as well as the variables of state, level of knowledge and stock of capital, to maximize the function of intertemporal utility. Finds that the growth of capital must be equal to the sum of the growth of the population and technological stocks.

The third model emphasizes international trade and the accumulation of human capital through learning in action as engines of economic growth.

But nevertheless; even when Lucas [7] states that in developing countries the elasticity of substitution between factors of production is less than 1, he develops his work based on Cobb-Douglas production functions which assume an elasticity of substitution equal to 1. In this paper we propose a production function with constant substitution elasticity since, in countries with emerging economies, it is very likely that it is less than unity.

In this second phase, the analysis of a model with endogenous technological change can be approached and finally the estimation of the model for when the economy is open. The first phase, which is summarized in numeral 2, was presented in 5th International Week of Science, Technology and Innovation [13]

### **Economic Growth Model with Exogenous Technological Change**

Like Lucas [7], it is considered a closed economy, with competitive markets, with identical rational agents, endowed with a technology with constant returns to scale. Let  $L(t)$  be the number of people (or, equivalently, the number of man-hours) willing to produce in a period of time  $t$ , with growth rate  $\eta = (L'(t))/(L(t))$  given exogenously.

$$\int_0^{\infty} U[c(t)]e^{-\rho t} dt = \int_0^{\infty} \frac{c(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \quad (1)$$

In equation 1 we include a utility function with elasticity of substitution  $(1/\sigma)$  between present consumption and constant future consumption, which allows us to describe the preferences among them. If  $\sigma$  is large, there is low elasticity between present and future consumption, this implies great preference for present consumption and low response of savings to the interest rate with high aversion to risk, which is to be expected in poor economies.

The production in a period  $t$ , is determined by the level of technology,  $A(t)$ , present in that period and by the levels of capital,  $K(t)$ , and work,  $L(t)$ , used in that period. Equation 2 describes the level of production, based on the variables described, using a production function with constant substitution elasticity, in which the relative share of each factor in the final product is determined by the distribution parameter  $\delta$  ( $0 \leq \delta \leq 1$ ) and the elasticity of substitution in production is given by  $\sigma_p = 1/(1+\alpha)$ , where  $\alpha$  is the substitution parameter ( $\alpha > -1, \alpha \neq 0$ )

$$Y(t) = A(t)[\delta K(t)^{-\alpha} + (1 - \delta)L(t)^{-\alpha}]^{-1/\alpha} \quad (2)$$

The elasticity of substitution is a local measure of substitution is a local measure of the substitution between the factors of production, that is, for a given level of production, measures the proportional change in the use of factors as a result of a proportional change in the marginal rate of labor substitution by capital (TMS),  $\sigma_p = [\text{TMS}/((K/L))] / [\partial \text{TMS} / \partial (K/L)]$ ; it can also be understood as a measure of the percentage change in the rate of use of the factors of production as a result of a percentage change in the relative prices of these factors. Thus, for high values of  $\sigma_p$ , the TMS responds intensely to changes in the relative prices of the factors and vice versa [14].

In this model it is considered that the technological change is given exogenously, then its growth rate  $\mu = (A'(t))/A(t)$  is given exogenously and the function that describes the trajectory of the technology over time is of the exponential type  $A(t) = A_0 e^{\mu t}$ . It is also assumed that the per capita production of the only good is divided into consumption and capital accumulation. Thus, the net national income is given by:  $Y(t) = c(t)L(t) + K'(t)$  where  $K'(t)$  is the net investment [ $(K'(t))/K(t)$  is the rate of change over time of the stock of capital] and since  $c(t)$  represents per capita consumption, the expression  $c(t)L(t)$  represents the level of national consumption.

The problem to solve consists of determining the trajectory in the time of the per capita consumption  $c(t)$ , that maximizes the function of utility given by the equations 1 and 2 incorporating the expression deduced on the income, which implies to solve the problem of control (equations 3 and 4), in which  $c(t)$  is the control variable and  $K(t)$  is the state variable:

$$\text{Max} \int_0^{\infty} \frac{c(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \quad (3)$$

$$\text{s. a. : } c(t)L(t) + \dot{K}(t) = A(t)[\delta K(t)^{-\alpha} + (1 - \delta)L(t)^{-\alpha}]^{-1/\alpha} \quad (4)$$

The solution to the corresponding Hamiltonian is obtained by applying the principle of maximization of Pontriagyn [14]

The optimal trajectory of capital and its exchange rate, , the rate of growth of capital is given by

$$\frac{\dot{K}(t)}{K(t)} = \eta + \frac{A(t)^\alpha [\mu - \dot{\varphi}(t)/\varphi(t)]}{A(t)^\alpha - \delta \varphi(t)^\alpha}$$

, which implies that the variation of capital is:

$$\frac{\dot{K}(t)}{K(t)} = \eta + \frac{\mu \sigma_p A_0^\alpha}{A_0^\alpha - \delta \varphi_0^\alpha e^{-\alpha \sigma_p \mu t}}$$

The growth rate of the economy is obtained from

$$G = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} + \frac{\dot{\varphi}(t)}{\varphi(t)}$$

and it is expressed as follows:

$$= (1 - \sigma_p)\mu + \eta + \frac{\mu\sigma_p A_0^\alpha}{A_0^\alpha - \delta\varphi_0^\alpha e^{-\alpha\sigma_p\mu t}}$$

Note that these rates are constant only in the case that the elasticity of substitution in production is 1.

It implies that three different cases must be considered (depending on the elasticity of substitution) in order to determine the rate at which per capita consumption must grow in order to satisfy the differential equation  $\dot{K}(t) = K(t)\varphi(t) - c(t)L(t)$  and stability of the model is achieved in the long term.

Note that when the elasticity of substitution is less than unity, the growth rate of capital is decreasing and tends to stabilize in the long term. If this were not the case and the rate of growth of capital would grow indefinitely, in developing economies there would be an unemployment problem since in these economies there is not enough saving to hook workers at certain prices, prices would be too high to the level of existing savings [13].

### **Economic Growth Model with Endogenous Technological Change**

When technological change is measured endogenously in the model, the growth rates of the economy, capital and consumption that allow trajectories that optimize the level of intertemporal utility are given in a similar way by the determinants of economic growth regardless of the level of elasticity in production substitution.

Romer [6] considers three premises that justify technological change as an engine of economic growth and in turn explain why population is not an appropriate measure for the level of human capital, which makes it necessary to include the level of schooling as well: (i) Technological change is at the basis of economic growth, provides a necessary incentive for capital accumulation, and together with capital is responsible for productivity; (ii) in a model with endogenous technological change, people contributing to its variation are motivated by market incentives; (iii) instructions for working with raw materials are inherently different from other economic goods.

In addition, it defines a rival good as one whose use by one person or company prevents its use by another person or company; otherwise, the good is not a rival. An asset is subject to exclusion if its owner can prevent others from using it. The model developed in the second part of this article treats technological change in an exogenous way, that is, it considers technology as an input that is both excludable and not rival. This model is compatible with the first premise (technological change drives growth) and the third premise (technology is a non-rival good) but is incompatible with the second premise (it does not consider the role of private maximizing behavior in the generation of technological change).

In order to introduce technological change endogenously to the model, an additional variable is required to measure the level of human capital by its level of schooling. Just as Lucas [7] considers, in the period of time  $t$ , a level  $L(t)$  of workers with a level of human capital or stock of

knowledge  $h(t)$ . By human capital we mean that a worker with level  $h(t)$  is productively equivalent to two workers each with level of human capital  $0.5h(t)$ , or a half worker with  $2h(t)$ .

Under the assumption that there are  $L$  workers in total, which have levels  $h$  ( $0 < h < \infty$ ) of knowledge, one has to  $L = \int_0^\infty L(h)dh$ . Now, if a worker devotes a fraction  $u(h)$  of his non-leisure time to production and the remaining fraction of his time,  $1-u(h)$ , to the accumulation of human capital then the effective labor force of production is  $L^c = \int_0^\infty [u(h)L(h)h]dh$  and the production function is given by  $Y = F(K, L^c)$ .

As a simplifying assumption it is assumed that all workers in the economy are identical, that is, have the same skill level and devote the same fraction of non-leisure time to production, in this case, the effective work force is  $L^c(t) = u(t)h(t)L(t)$  and therefore the CES production function to consider is  $Y(t) = A\{\delta K(t)^{-\alpha} + (1-\delta)[u(t)h(t)L(t)]^{-\alpha}\}^{-1/\alpha}$ ; however, since  $A$  is constant,  $A=1$  can be assumed without loss of generality in the results.

On the other hand, the effort  $1-u(t)$  destined to the accumulation of human capital is related to its growth rate, therefore it is possible to affirm that the rate at which the designs that increase the ability,  $h$ , of the workers are generated is given by:

$$\dot{h}(t) = h(t)^\xi M[1 - u(t)],$$

where  $M$  is a growing function with  $M(0)=0$ .

According to Usawa [12] the evolution of  $h(t)$  is determined by the allocation of resources between a research sector and a final goods sector; he proposes that the equation for defining  $\dot{h}(t)$  be linear in  $h(t)$  and therefore the level of human capital grows exponentially and grows steadily if the human effort devoted to research remains constant. Lukas [7] uses this approach and proposes that the growth rate of human capital is:

$$\frac{\dot{h}(t)}{h(t)} = \xi[1 - u(t)].$$

The objective now is to determine the trajectories that the per capita consumption  $c(t)$  and the production effort  $u(t)$  must follow over time in order to maximize the intertemporal utility function at all times. That is, the following problem must be solved:

$$\text{Max} \int_0^\infty \frac{c(t)^{1-\sigma}}{1-\sigma} L(t) e^{-\rho t} dt \quad (5)$$

$$\text{s. a. : } L(t)c(t) + \dot{K}(t) = \{\delta K(t)^{-\alpha} + (1-\delta)[u(t)h(t)L(t)]^{-\alpha}\}^{-1/\alpha}$$

(6)

$$\dot{h}(t) = \xi[1 - u(t)]h(t) \quad (7)$$

in which  $c(t)$  and  $u(t)$  are the control variables and  $K(t)$  and  $h(t)$  are the state variables.

As in the previous case, the Pontriagyn maximization principle is applied and it is obtained that the optimal solution must satisfy the following necessary conditions:

In the margin, goods must be valued equally in their two uses: consumption and capital accumulation. Time must also be valued in its two uses: production and capital accumulation. The exchange rate of the efficiency price of physical capital must be equal to the discount rate minus the marginal productivity of capital.

The system of simultaneous non-linear differential equations must be given regularity

conditions in order to obtain results consistent with economic theory. Under the assumption that the growth rate of per capita consumption is constant over time, it is obtained that the optimal trajectory of the product to capital ratio is constant over time, which implies that the optimal requires that the growth rate of the economy be equal to the growth rate of capital. The optimal capital trajectory must grow at a constant rate given by the sum of the growth rates of labour force and human capital. In the long run, human capital must grow at the same rate as the economy. The growth of per capita consumption is equal to the growth of technology minus the discount rate plus the growth rate of the labor force, multiplied by the rate of substitution between present consumption and future consumption; which implies that the system grows more to the extent that there is more preference for future consumption.

### Fundamentals of Economics-Mathematics

The production of a country in time period  $t$  is determined by the level of technology,  $A(t)$ , present in that time period and by the levels of capital and labour used in that period. The production level is described by equation (2) which represents a production function with constant substitution elasticity, in which the relative share of each factor in the final product is measured by the substitution parameter  $\delta$  ( $0 < \delta < 1$ ); and the substitution elasticity in production  $\sigma_p = 1/(1+\alpha)$ , where  $\alpha$  is the substitution parameter ( $\alpha > -1, \alpha \neq 0$ ).

Substitution elasticity  $\sigma_p$  is a local measure of the substitution between the two factors of production. Specifically, for a given level of production,  $\sigma_p$  measures the proportional change in the factor utilization rate as a result of a proportional change in the marginal rate of capital-labour substitution (MRS).

$$\sigma_p = \frac{\Delta(K/L)/K/L}{\Delta MRS/MRS} = \frac{MRS/(K/L)}{\partial MRS/\partial(K/L)} \quad (8)$$

Substitution elasticity can also be understood as a measure of the percentage change in the rate of use of factors of production as a result of a percentage change in the relative prices of those factors. Thus, for high values of  $\sigma_p$ , the MRS responds intensely to changes in relative factor prices and vice-versa.

The curvature of isocuantas (contour lines) is measured by the elasticity of substitution, presenting five cases:

**First case:** in the limit when  $\alpha \rightarrow \infty$  you have to  $\sigma_p \rightarrow 0$  and in this case the

$$MRS = - \frac{\partial L(t)}{\partial K(t)} = \frac{\delta}{1-\delta} \left[ \frac{L(t)}{K(t)} \right]^{1+\alpha}$$

tends to zero if  $K(t) > L(t)$  or tends to infinity if  $K(t) < L(t)$ . In this borderline case, the situation is impossible and therefore the curvature of the isocuantas appears at right angles. The CES production function tends to be a Leontief production function.

**Second case:** when  $\alpha > 0$  you have to  $0 < \sigma_p < 1$ . The isoquantas are in the form

$$\delta K(t)^{-\alpha} + (1 - \delta)L(t)^{-\alpha} = \left[ \frac{Y(t)}{A(t)} \right]^{-\alpha}$$

In the equation of isoquantas, the term on the right is constant in an instant of time  $t$ . Note that if  $K(t) \rightarrow 0$ , it is obtained that  $\delta K(t)^{-\alpha} \rightarrow \infty$ ; therefore  $K(t)$  and  $L(t)$  cannot be zero. The isoquantas are decreasing and convex with asymptotes given by

$$K(t) = \delta^{1/\alpha} \frac{Y(t)}{A(t)} \text{ and } L(t) = (1 - \delta)^{1/\alpha} \frac{Y(t)}{A(t)}.$$

**Third case:** when  $\alpha \rightarrow 0$  you have to  $\sigma_p \rightarrow 1$ . The CES production function tends to be Cobb-Douglas type with substitution elasticity 1. To see this, write the CES production function as follows:

$$\frac{Y(t)}{A(t)} = [\delta K(t)^{-\alpha} + (1 - \delta)L(t)^{-\alpha}]^{-1/\alpha}$$

Taking logarithms on both sides of the equation and applying limit when  $\alpha \rightarrow 0$  is obtained:

$$\lim_{\alpha \rightarrow 0} \ln \left[ \frac{Y(t)}{A(t)} \right] = \lim_{\alpha \rightarrow 0} \frac{-\ln[\delta K(t)^{-\alpha} + (1 - \delta)L(t)^{-\alpha}]}{\alpha}$$

Apply the rule of L'Hôpital to the expression on the right to get:

$$\lim_{\alpha \rightarrow 0} \ln \left[ \frac{Y(t)}{A(t)} \right] = \lim_{\alpha \rightarrow 0} \frac{-[-\delta K(t)^{-\alpha} \ln K(t) - (1 - \delta)L(t)^{-\alpha} \ln L(t)]}{\delta K(t)^{-\alpha} + (1 - \delta)L(t)^{-\alpha}}$$

$$\lim_{\alpha \rightarrow 0} \ln \left[ \frac{Y(t)}{A(t)} \right] = \frac{\delta \ln K(t) + (1 - \delta) \ln L(t)}{1} = \ln [K(t)^\delta L(t)^{(1-\delta)}]$$

So, when  $\alpha \rightarrow 0$  you have to:  $\ln \left[ \frac{Y(t)}{A(t)} \right] \rightarrow \ln [K(t)^\delta L(t)^{(1-\delta)}]$  and therefore,  $Y(t) \rightarrow A(t) K(t)^\delta L(t)^{(1-\delta)}$

It is thus demonstrated that when  $\alpha \rightarrow 0$  the CES production function effectively tends to a Cobb-Douglas

**Fourth case:** When  $-1 < \alpha < 0$  you have to  $\sigma_p > 1$ . In the equation of isoquantas

$$\delta K(t)^{-\alpha} + (1 - \delta)L(t)^{1-\alpha} = \left[ \frac{Y(t)}{A(t)} \right]^{-\alpha}$$

the exponents of the terms on the left are positive. Therefore, the isoquantas cut both axes. That is, when

$$K(t) = 0, L(t) = (1 - \delta)^{1/\alpha} \frac{Y(t)}{A(t)} \text{ and when } L(t) = 0, K(t) = \delta^{1/\alpha} \frac{Y(t)}{A(t)}.$$

**Fifth case:** when  $\alpha \rightarrow -1$  one has to  $\sigma_p \rightarrow \infty$ ; in the limit, both exponents of the isoquantas are one. The isoquantas are straight lines. In this case, the production factors are perfect substitutes.

## CONCLUSIONS

When the elasticity of substitution in production is greater than 1, the economy will experience high savings rates that in turn push up the interest rate, which leads to a deterioration in the distribution of income causing the rich to appropriate each time a greater proportion of income than the poor. There is also a decrease in the consumption to capital ratio, that is, over time a decrease in the level of consumption is experienced with respect to the level of capital present in the economy.

Conversely, when the elasticity of substitution in production is less than unity, the saving rates will be increasingly low, which leads to an economy in which growth does not depend on savings and therefore does not worsen the distribution of income. The result obtained here indicates that saving is not a determinant of growth, this is due in turn to the assumption of full employment in the model. If this assumption were removed, saving would become one of the determinants of growth, and in this case, the consumption to capital ratio increases with the passage of time leading to the level of capital present in the economy generates more and more of national consumption.

In a model with endogenous technological change, if the level of schooling of workers increases, the economy will grow at a higher rate than the population, thus achieving higher levels of output per worker. It is enough for workers to specialize more and more in the generation of designs that facilitate production.

When technological change is endogenous, there are two possibilities: whether growth is determined by technology or whether it is determined by capital. If a capital-led model is desired, higher savings rates and hence higher interest rates are required, a low interest rate would determine capital growth below full employment; therefore, a capital-led model is conditional on high interest rates causing problems in income distribution. Now, a technology-driven model implies a better wage as more technology becomes available; if technology increases, the marginal productivities of capital and labor increase, which implies higher levels of capital and labor and, therefore, higher economic growth, which leads to a better distribution of income.

The results obtained in the development of the model with endogenous technological change are similar to those obtained by Lukas [7]. Therefore, when the technological change is endogenous to the model, it does not matter the level of elasticity of substitution that is had in the production in a country, because always the rates of growth of the economy, of the capital and of the consumption are going to be given in the same way by the determinants of the growth.

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